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Circular and Elliptical Geometry- Mahāvīra's Contributions

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Abstract : While Indian mathematics is often celebrated for arithmetic and algebra, its foundation lies in Kṣetragaṇita (Geometry). This paper explores how Jaina mathematicians, particularly Mahāvīra, integrated geometric spatial logic into a rigorous mathematical system.

The study focuses on three primary areas:

- Circular Geometry: Comparing Mahāvīra's practical approximations with his highly accurate calculations found in the *Dhavalā* which achieves a remarkable 0.003% error rate.
- Segments and Bow-shapes: Analyzing the area of segments (like the *yava* grain) using the chord (*jīva*) and arrow (*bāṇa*). The paper evaluates Mahāvīra's formula against Śrīdhara's methods and modern trigonometric results.
- The Elongated Circle (Āyatavṛtta): Investigating the Jaina treatment of the ellipse. By comparing Mahāvīra's circumference and area formulas to modern elliptical geometry, the paper demonstrates that the *Āyatavṛtta* and the modern ellipse are conceptually nearly identical.

Introduction: Although Indian mathematical thought is frequently associated with breakthroughs in arithmetic and algebra, its core inspiration and foundation are undeniably rooted in geometry, or Kṣetragaṇita. The origins of algebra in India can be traced back to the constructional geometry of the *Śulbasūtra* period, where algebraic truths were often expressed and validated through geometric forms. This spatial perspective was so pervasive that even basic arithmetic operations, like multiplication and division, were explained through geometric interpretations. Similarly, trigonometry was treated as the geometry of the triangle, while the theory of numbers evolved from the study of right-angled triangles and rational rectilinear figures. Ultimately, this geometric grounding provided a visual framework that supported the development of India's sophisticated computational systems. Distinguished Jaina mathematicians, such as Mahāvīra, Vīrasena, Yativṛṣabha, and Śrīdhara, demonstrated remarkable skill in calculating the areas and volumes of both simple and highly sophisticated figures. Their geometric inquiries expanded beyond basic polygons to include intricate triangular and quadrilateral shapes, as well as complex circular and ring-shaped forms. By mastering the combinations of these diverse figures, they applied their mathematical talents to solve advanced spatial problems that spanned both practical engineering and complex cosmological modeling.The most important curvilinear curve dealt by jainas is circle.

Circle:

In GSS V. 7-19, p.435 Mahāvi¯ra gives the approximate area of a circle [1]

The above sloka mentions that for a circle the measure of the diameter multiplied by 3 is the measure of the mcircumference and the number representing the square of half the diameter multiplied by 3 is the area.

Which means that approximate area = $3 \times \left(\frac{d}{2}\right)^2$ and approximate perimeter = 3x diameter. Here the value of π is 3.

As per GSS 7-60 the minutely accurate value of circumference[2] $C=\sqrt{10}d$ and area = $C\times\frac{d}{4}=\sqrt{10}\left(\frac{d}{2}\right)^2$.

Also in Dhavalā 4[3] p. 13 we see $C=\frac{355d}{113}$. Here putting d = 1, circumference = 3.141.

But **modern value = 3.142**. So the **error is .003%**

According to Mahāvīra circumference = $\sqrt{10}=3.146.$

Segment :

In GSS *op-cit* v. 7-70 ½ , p.467[4], Mahāvīra gives a method to find the area of a segment

The verse means that the measure of the string (chord) multiplied by one –fourth of the measure of the arrow and then multiplied by the square root of 10 gives the accurate area of a figure having the out line of a bow and also in the case of a figure resembling the longitudinal section of a yava grain.

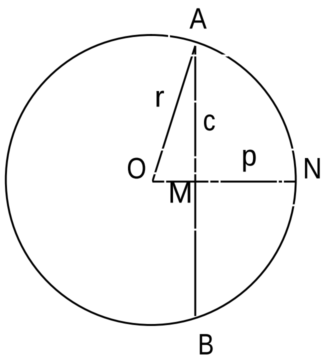

Fig. 7

jīva = AB = c, bāṇa = MN = p

According to Mahāvīra

Area of segment $ANB=\sqrt{10}\,\frac{\text{jīva} \times \text{bāṇa}}{4}$

$$=\sqrt{10}\; c.\frac{p}{4}.$$

But according to Śrīdhara[5]

Area of segment ANB = $\sqrt{\frac{10}{9}\left[\frac{(\text{jīva}+\text{bāṇa})}{2}\right]^2} \times \text{bāṇa}$

$= \sqrt{\frac{10}{9}\left[\frac{(c+p)}{2}\times p\right]^2}$

By the modern mathematics area of ANB =

$r^2\left[\cos^{-1}\left(\frac{r-p}{r}\right)-\frac{c}{2}(r-p)\right]$, r (radius) $=\frac{c^2+4p^2}{8p}$

e.g., p = 2 c = 5

According to Mahāvīra Area = $\sqrt{10}\; c\times\frac{p}{4}$

$$=\sqrt{10}\times 5.\frac{2}{4}=7.91$$

According to Śrīdhara, Area

$$=\sqrt{\frac{10}{9}\left(\frac{c+p}{2}\times p\right)^2}=\sqrt{\frac{10}{9}\left(\frac{5+2}{2}\times 2\right)^2}$$

$$=\frac{\sqrt{10}}{3}\times\frac{5+2}{2}\times 2 = 7.38$$

According to modern mathematics

$$\text{Area} = r^2\cos^{-1}\left(\frac{r-p}{r}\right)-\frac{c}{2}(r-p)$$

Where $r=\frac{c^2+4p^2}{8p}=\frac{25+4\times 4}{8\times 2}=2.56$

$$\therefore \text{Area} = (2.56)^2\cos^{-1}\left(\frac{2.56-2}{2.56}\right)-\frac{5}{2}(2.56-2)$$

$$=6.56\times(77.3)^0-1.4=6.56\times 77.3\times\frac{\pi}{180}-1.4$$

= 7.45

So Śrīdhara is closer to modern value.

Āyatavṛtta (Elongated circle)

G.S.S. v.7.21 gives the rules for area and circumference of 'Āyatavṛtta' (elongated circle similar to ellipse).

व्यासार्धयुतो द्विगुणित आयतवृत्तस्य परिधिरायामः ।

विष्कम्भचतुर्भागः परिवेषहतो भवेत्सारम् ।।

The above stanza means that the larger diameter increased by half of the shorter diameter and multiplied by two gives the measure of the circumference of the elongated circle. Also one-fourth of the shorter diameter multiplied by the circumference gives the approximate area. ie

Circumference C = (2a+b)2, where 2a is the major axis and 2b is the minor axis and Area = $\frac{1}{4}2bC$ where C is the circumference.

By the formula given above area of the elongated circle is $2ab+b^2$. But in modern mathematics the area of an ellipse is πab. If π is taken as 3 then area = 3ab. So the elongated circle of G.S.S and ellipse of modern mathematics are almost similar figures

Conclusion: The mathematical contributions of Mahāvīra demonstrate that ancient Indian geometry was far more than a set of practical rules; it was a rigorous, visual system that bridged the gap between physical measurement and abstract logic. By refining the properties of the circle, providing early approximations for the ellipse (*Āyatavṛtta*), and developing formulas for complex segments, Mahāvīra and his Jaina contemporaries laid a critical foundation for later advancements in trigonometry and calculus. Their ability to achieve a 0.003% error rate in circular calculations proves that their spatial reasoning.

References

1 Padmavathamma, Ganita Sara sangraha of Mahavira- with translation, Sri Siddhantakirthi Granthamala, Shimoga, Karnataka 2000

त्रिगुणीकृतविष्कम्भः परिधिव्र्यासार्धवर्गराशिरयम्।

त्रिगुणः फलं समेऽर्धे वृत्तेऽर्धं प्राहुराचार्याः**

2 *Ibid.,* v.7-60, p. 460.

3 Dr. Hiralal jain (ed.), Dhaval¡3, Hindi translation and notes of Dhavala commentaries of Vi¯rasena (Ji¯varaj jain Granthamala), Jain Samskriti samrak ṣaka sanga,Sholapur- 2, 2002, p.249.

4 इषुपादगुणश्च गुणो दशपदगुणितश्च भवति गणितफलम् ।

यवसंस्थानक्षेत्रे धनुराकारे च विज्ञेयम् **

5 Śri¯dhara Trisatika v. 47 Department of Mathematics and Astronomy Lucknow, 1959 *Ibid.,v* 7.21, p.436