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Some significant contributions of Mahāvi¯ra - the great Jaina Mathematician

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Introduction: Jaina philosophy is very rich in Mathematics. Many Jaina philosophers were very good mathematicians too. Śri¯dhara, , Vi¯rasena, Mahāvi¯ra, Yativṛṣabha are a few among them.Mahavira (or Mahaviracharya), a 9th-century Jain mathematician from Karnataka, holds a pioneering position in the history of Indian Mathematics. Mahāvi¯ra has the authorship of the pure mathematical text Gaṇita Sāra Saṅgraha (G.S.S). Many new mathematical concepts were discussed in G.S.S. Mahāvi¯ra introduced these concepts in the form of very interesting queries and discussion. His approach to the mathematical concepts are different from that of the modern mathematics

Combinations

In G.S.S. 6-218 Mahāvi¯ra gives the general formula for the number of combinations of 'n' things taken 'r' at a time[1]This sloka means that write the numbers starting from 1 upto the given number in one line. Below that write the same numbers in the reverse order. The product of 1 2 3 or more numbers in the upper row taken from right to left be divided by the corresponding product of 1 2 3 or more numbers in the lower row also taken from right to left is the required combination in each case.

Eg:- Let the number of things be 5. Then write the numbers from 1 to 5 and then 5 to 1 as follows

1 2 3 4 5
5 4 3 2 1

By the rule

$$5C_1 = \frac{5}{1} = 5 \qquad 5C_2 = \frac{5\times4}{1\times2} = 10$$

$$5C_3 = \frac{5\times4\times3}{1\times2\times3} = 10 \qquad 5C_4 = \frac{5\times4\times3\times2}{1\times2\times3\times4} = 5$$

$$5C_5 = \frac{5\times4\times3\times2\times1}{1\times2\times3\times4\times5} = 1$$

When talking about the different types of fractions in G.S.S 3.138 Mahāvi¯ra says that there are 26 types of Bhāgamātrā fractions (combinations of five simple varieties of fractions). This number 26 is obtained as follows:-

$5C_2 + 5C_3 + 5C_4 + 5C_5$ = 10+10+5+1 =26

But the credit of the formula for permutations and combinations is given to Herigon (1634)[2] who lived about 800 years after the period of Mahāvi¯ra.

Unit Fractions

Another important contribution of Mahāvi¯ra is the idea about unit fractions. The word used by him to represent unit fraction is 'rūpāmsʼaka rāsʼi'. He was very much interested in problems connected with unit fractions. Some examples are the following.

a. Mahāvi¯ra gives the method to decompose a unit fraction into 2 unit fractions v 3.85, p 124 .[3]

This sloka means that the denominator of the given sum multiplied by a chosen number is the denominator of one of the intended fractions and this denominator divided by the previously chosen number as lessened by one gives the other denominator, or factorise the given denominator and multiply one of these factors and the sum of the factors. This is the first required denominator. Similarly multiplying the other factor and the sum of the factors will give the second denominator.

ie., if $\frac{1}{n}$ is the given fraction where p is any chosen quantity. $\frac{1}{n} = \frac{1}{np} + \frac{1}{\frac{np}{p-1}}$

Or if $\frac{1}{n} = \frac{1}{ab}$, $b \geq 1$ then. $\frac{1}{n} = \frac{1}{a(a+b)} + \frac{1}{b(a+b)}$

b. In G.S.S v.3.78, p.119 Mahāvi¯ra gives the method to decompose a unit fraction as the sum of fractions with given numerators:[4]

By this sloka if $\frac{1}{n}$ is the sum of the given fractions and a, b, c, d are the numerators then

$$\frac{1}{n} = \frac{a}{n(n+a)} + \frac{b}{(n+a)(n+a+b)} + \frac{c}{(n+a+b)(n+a+b+c)} + \frac{d}{d(n+a+b+c)}$$

$$RHS = \frac{a(n+a+b)+bn}{n(n+a)(n+a+b)} + \frac{c+n+a+b}{(n+a+b)(n+a+b+c)}$$

$$= \frac{a+b}{n(n+a+b)} + \frac{1}{(n+a+b)} = \frac{n+a+b}{n(n+a+b)} = \frac{1}{n} = LHS$$

So the result is true.

c. In G.S.S. v.3.75, p.116 he also gives the expression for '1' as the sum of 'n' unit fractions.[5]

As per the sloka if the sum of n unit fractions is one then the denominators of the fractions are 2x1, 3, 3^2, 3^3, ...3^{n-2}, $2x3^{n-1}$

ie., $1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots \ldots . \frac{1}{3^{n-2}} + \frac{3}{23^{n-1}}$

d. Expression for '1' as the sum of '2n' unit fractions is given in G.S.S v.3.77, p.118[6]

This sloka states that if the sum of different unit fractions is 1 then the denominators are obtained by multiplying one integer (say n) and the next integer (n+1) and halving the product. The integer n can take values beginning from 2.

$$1 = \frac{1}{2.3.\frac{1}{2}} + \frac{1}{3.4.\frac{1}{2}} + \frac{1}{4.5.\frac{1}{2}} + \cdots .. \frac{1}{(2n-1).2n.\frac{1}{2}} + \frac{1}{2n.\frac{1}{2}}$$

For finding the sum and difference of simple fraction the 1st step given by Mahāvi¯ra is to make the denominators equal. The fractions of equal denominators can be added or subtracted by doing the operation for the numerators and putting the common denominator.

Least Common Multiple

Mahāvi¯ra introduced the word Niruddha means L.C.M in G.S.S. v.3.56, p.108[7]

As per the sloka niruddha is obtained by means of the continued multiplication of all the common factors of the denominators and all their ultimate quotients. This is modern method to find the L.C.M.

Geometry

The 7th chapter of GSS is completely given for geometry. In this chapter Mahāvi¯ra discusses different geometrical shapes and the formula to find their area. आयतवृत्त is one among these which represent an elongated circle (similar to ellipse).

In GSS 7.21 the approximate area of an elongated circle is given[8]. This stanza means that the larger diameter increased by half of the shorter diameter and multiplied by two gives the measure of the circumference of the elongated circle. Also one-fourth of the shorter diameter multiplied by the circumference gives the approximate area. ie Area = $\frac{1}{4}2b.C$ where C is the circumference and $C = (2a + b)2$ where 2a is the major axis and 2b is the minor axis.

,

By the formula given above **area = 2ab+b²** . But by modern mathematics the area of an ellipse is πab. If π is taken as 3 then area = 3ab.

Also the rule for arriving at the minutely accurate values of circumference and area is given in GSS7.63 [9].This stanza means that the square of the short diameter multiplied by six and the square of twice the longer diameter is added to this. The square root of this sum gives the measure of the circumference and this circumference multiplied by one-fourth of the shorter diameter gives the minutely accurate area of an elliptical figure. That means

Circumference $C = \sqrt{(2b)^2 6 + (4a)^2}$ and Area = $C.\frac{1}{4}2b$. But by the modern mathematics accurate perimeter is obtained by $\int_0^{\pi}(a^2\sin^2\vartheta + b^2\cos^2\vartheta)^{\frac{1}{2}}d\vartheta$

Even Mahāvi¯ra has given only the idea of ellipse but has not clearly completed the properties of ellipse.

In addition to these, Mahāvi¯ra was the first Indian Mathematician to discuss the following shapes.

a. Yavākāra b. Murjākāra c. Paṇavākāra d. Vajrākāra

e Ubhayaniṣedha kṣetra f. Ekaniṣedha kṣetra
g. Hatadanta kṣetra h. Ṣaṭbhuja kṣetra

Jaina mathematician Mahāvi¯ra, has also applied his mathematical talents to find out the areas, volumes etc. of not only simple figures and objects but also of complex ones. They include triangular shapes, quadrilateral shapes, circular and ring shapes and combinations of these.

Conclusion: The works of Mahāvīra reveal a mathematician who was centuries ahead of his time, particularly in the fields of combinatorics and number theory. By formalizing the general formula for combinations nCr eight centuries before European mathematicians and developing systematic methods for unit fractions and L.C.M. (Niruddha), he transformed Indian mathematics into a rigorous analytical science. Furthermore, his ability to categorize complex shapes—ranging from the elliptical Āyatavṛtta to the drum-shaped Paṇavākāra—shows a unique mastery of spatial logic that balanced practical engineering with high-level theory**.**

References

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एकाद्येकोत्तरतः पदमूर्ध्वाधर्यतः क्रमोत्क्रमशः

स्थाप्य प्रतिलोमघ्नं प्रतिलोमघ्नेन भाजितं सारम्

2 D.E. Smith, History of Mathematics, Vol. 2. Ginn and company, p. 527 1925

[3] G.S.S *opcit.*, v. 3.85, p. 124

वाञ्छाहतयुतिहारश्छेदः स व्येकवाञ्छयाप्तोन्यः।

फलहारहारलब्धे स्वयोगगुणिते हरौ वा स्तः॥

[4] *Ibid.*, v.3.78, p.119

लब्धहरः प्रथमस्यच्छेदः सस्वांशकोऽयमपरस्य

प्राक् स्वपरेण हतोऽन्त्यः स्वांशेनैकांशके योगे॥

[5] *Ibid.*, v.3.75, p.116

रूपांशकराशीनां रूपाद्यास्त्रिगुणिता हराः क्रमशः।

द्विद्वित्र्यांशाभ्यास्तावादिमचरमौ फले रूपे॥

[6] *Ibid.*, v.3.77, p.118

एकांशकराशीनां द्याद्या रूपोत्तरा भवन्ति हराः।

स्वासन्नपराभ्यस्तास्सर्वे दलिताः फले रुपे॥

[7] *Ibid.*, v.3.56, p.108

छेदापवर्तकानां लब्धानां चाहतो निरुद्ध स्यात्

हरहतनिरुद्धगुणिते हरांशगुणे समो हारः

[8] *Ibid.,v* 7.21, p.436

व्यासार्धयुतो द्विगुणित आयतवृत्तस्य परिधिरायामः ।

विष्कम्भचतुर्भागः परिवेषहतो भवेत्सारम् ।।

[9] *Ibid.,v* 7.63, p.461

व्यासकृतिषड्गुणिता द्विसङ्गुणायामकृतियुता (पदं) परिधिः।

व्यासचतुर्भागगुणश्चायतवृत्तस्य सूक्ष्मफलम् ।।