National Journal of
Hindi & Sanskrit Research

ISSN: 2454-9177
NJHSR 2019; 1(22): 118-122

www.sanskritarticle.com

Dr. P.M. Mini,
Assistant Professor and Head,
Department of Mathematics,
M.A.M.O.College Mukkom,
Kerala

Correspondence:
Dr. P.M. Mini,
Assistant Professor and Head,
Department of Mathematics,
M.A.M.O.College Mukkom,
Kerala

An Exploaration of First-Degree Equations in Mahāvīrācārya's G.S.S.

Dr. P.M. Mini

Abstract: Ancient Indian mathematicians made profound contributions to algebra, with Jaina philosopher-mathematicians significantly enriching areas such as equations and series. Among them, the eminent Mahāvīrācārya stands out for his innovative pedagogical approach, often using engaging puzzles to transmit mathematical principles to his disciples. In the *Gaṇita Sāra Saṅgraha* (G.S.S.), Mahāvīra details various equation types and their solutions—methods that remain highly relevant for modern students seeking a deeper conceptual grasp of mathematics. This article provides a critical analysis of his algebraic techniques and their educational utility."

Keywords: Linear equation

Introduction : A. L. Basham observed that while Greek mathematics was primarily rooted in geometry, ancient India transcended these boundaries through a sophisticated numeral system. This gave rise to an algebra that enabled calculations far more complex than those of the Greeks, fostering a study of numbers for their own sake. Within this historical framework, Jaina mathematicians such as Śrīdhara, Vīrasena, and Mahāvīra made foundational contributions. Mahāvīra's *Gaṇita Sāra Saṅgraha* (G.S.S.) is particularly striking for its imaginative problems, which possess a poetic elegance that can captivate any reader. For educators, his methods provide a vital resource to transform mathematics from a tedious task into an engaging and accessible subject. While the G.S.S. addresses everything from quadratic systems to multi-variable algebra, this section focuses specifically on Mahāvīra's solutions for linear equations in one unknown.

Linear equations in one unknown:

A linear equation in a single unknown is defined as an equation of the first degree, such as 2x-1=3, 5y+6=2, 2x=7 or 7y=2. Within the *Gaṇita Sāra Saṅgraha* (G.S.S. v. 3.107)[1], Mahāvīra introduces the methodology for solving these equations. Interestingly, he embeds these algebraic solutions within his discussion on the simplification of complex and compound fractions, showcasing the integrated nature of Jaina mathematical logic..

The above mentioned method is that, put '1' in the place of the unknown and find the result. Divide the result given in the problem, by the result obtained by putting the unknown equal to 1. The quotient will be the value of the unknown.

Mahāvi¯ra explains the above method by an example:-

Eg[2]:- The sum of $\frac{1}{8}$ of one unknown, $\frac{1}{3}$ of $\frac{1}{4}$ of the unknown, $\frac{1}{2}$ of $\frac{1}{5}$ of the unknown and $\frac{1}{6}$ of $\frac{3}{4}$ of $\frac{1}{5}$ of the unknown is $\frac{1}{2}$. What is the unknown?"

The given problem is:-

$$\frac{1}{8}x+\frac{1}{3}.\frac{1}{4}x+\frac{1}{2}.\frac{1}{5}x+\frac{1}{6}.\frac{3}{4}.\frac{1}{5}x=\frac{1}{2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

what is *x*, the unknown?
Putting x=1 in the Left Hand Side of equation (1) it becomes

$$\frac{1}{8}+\frac{1}{3}.\frac{1}{4}+\frac{1}{2}.\frac{1}{5}+\frac{1}{6}.\frac{3}{4}.\frac{1}{5}=\frac{1}{8}+\frac{1}{12}+\frac{1}{10}+\frac{1}{40}=\frac{40}{120}=\frac{1}{3}$$

The given sum is $\frac{1}{2}$ ∴ By the above given rule, the unknown. $=\frac{\frac{1}{2}}{\frac{1}{3}}=\frac{3}{2}$

When talking about miscellaneous problems on fractions in GSS 4.7, Mahāvi¯ra projects a beautiful problem in one unknown[3] . A śrāvaka having gathered lotuses and loudly uttering hundreds of prayers offered $\frac{1}{3}$ of those lotuses to the 1^{st} ti¯rthankarā and $\frac{1}{3}$, $\frac{1}{4}$and $\frac{1}{6}$of this (i.e., $\frac{1}{3}$) respectively to the other 3 ti¯rthankarās. $\frac{1}{9}$as well as $\frac{1}{12}$ of this $\frac{1}{3}$to another ti¯rthankarā and the remaining 19 ti¯rthankaras were offered 2 each. Then what is the total number of lotuses.
In order to solve such problems Mahāvi¯ra states a rule in GSS.[4]
If certain specified fractional quantities of a total quantities are removed from the total quantity, Mahāvi¯ra gives the rule to find the total quantity when the numerical value of the removed portion is given. According to the above rule divide dṛśya by (1- bhāga) to get the required quantity.
The unknown total quantity $x=\frac{a}{1-b}$ where 'a' is ' dṛśya', the remaining quantity and 'b' is 'bhāga' the sum of the fractional quantities.
In the above problem the offerings to the various ti¯rthankaras can be accounted as follows:

1^{st} ti¯rthankarā = $\frac{1}{3}$ = $\frac{1}{3}$

2^{nd} ti¯rthankarā = $\frac{1}{3}.\frac{1}{3}$ = $\frac{1}{9}$

3^{rd} ti¯rthankarā = $\frac{1}{3}.\frac{1}{4}$ = $\frac{1}{12}$

4^{th} ti¯rthankarā = $\frac{1}{3}.\frac{1}{6}$ = $\frac{1}{18}$

5^{th} ti¯rthankarā = $\frac{1}{3}.\frac{1}{9}+\frac{1}{3}.\frac{1}{12}$ = $\frac{1}{27}+\frac{1}{36}$

∴ b = sum of fractional quantities

$$=\frac{1}{3}+\frac{1}{9}+\frac{1}{12}+\frac{1}{18}+\frac{1}{27}+\frac{1}{36}=\frac{1}{108}$$

The number of remaining lotuses = $19\times 2=38$
∴ a = remaining quantity = 38

∴ the total number of lotuses = unknown = $x=\frac{a}{1-b}=\frac{38}{1-\frac{70}{108}}=\frac{38}{\frac{38}{108}}=$ 108

It is obvious that this is derivable from the equation $x-bx=a$

Linear equations in two or more unknowns:

A linear equation in two unknown or variables is an equation of degree 1.
Example 2x-3y=3 , 5y+2=6 etc
Mahāvi¯ra has highlighted the method to solve 1^{st} degree equations in 2 unknowns.[5]
As per the sloka to separate the prices of an interchangeable larger and smaller number of two different things from the given mixed sums of the prices, first multiply the higher price sum by the larger number. From this subtract the lower price number as multiplied by the smaller number. Then divide the result by the difference between the squares of the numbers relating to these things. This will give the price of thing which is larger in number. The other is obtained by interchanging the multipliers.
Here Mahāvi¯ra gives the rule for the solution of system of equations of the form
ax+by=m................ (1) and bx+ay=n................ (2)
as $x=\frac{am-bn}{a^2-b^2}$ and $y=\frac{an-bm}{a^2-b^2}$
The above rule is true as per modern mathematics

In order to verify the above rule he has given an example:
The mixed price of 9 citrons and 7 fragrant wood apples is 107 and the mixed price of 7 citrons and 9 fragment wood apples is 101. What is the price of a citron and of a wood apple?
Let the price of a citron = x
Let the price of a wood apple = y
Then given $9x+7y = 107$
$7x+9y = 101$

By the rule $x = \frac{9\text{x}107-7\text{x}101}{9^2-7^2} = \frac{963-707}{31} = 8$

$y = \frac{9\text{x}101-7\text{x}107}{9^2-7^2} = \frac{160}{32} = 5$

In G.S.S. 6.270-272 ½ Mahāvi¯ra describes a problem[6]-
A greatman with magical powers seeing a cock fight speaks individually to both the owners of the cock. He says to one 'if your bird wins, then you give me your stake-money but if you don't win, I shall pay you 2/3 of it'. At the same time going to the other owner he promises in the same way to give 3/4 if he loses. Tell me the mathematician the stake-money of each of the cock-owners if the man earns a profit of 12 gold coins.
This problem can be written algebraically as

If $x - \frac{3}{4}y = 12$ and $y - \frac{2}{3}x = 12$ then x=? and y=?

Then Mahāvi¯ra gives the solution[7] for problems like

$$x - \frac{c}{d}y = p \quad \text{and} \quad y - \frac{a}{b}x = p$$

as

$$x = \frac{(c+d)b}{(c+d)b-(a+b)c} \text{X}\, p \qquad \text{and} \qquad y = \frac{(a+b)d}{(a+b)d-(c+d)a} \text{X}\, p$$

where x and y are the moneys on hand with the gamblers and $\frac{a}{b}$ and $\frac{c}{d}$ the fractional parts taken from them and p is the gain.
In GSS 6.160-162 Mahāvi¯ra puts forward an interesting problem.[8] Four merchants who had invested their money in common were asked by the customs officer, each separately, what the value of the commodity they were dealing in was and one merchant among them deducting his own investment said that the value was in fact 22. Then another said that it was 23 and another 24 and the fourth 27. What is the value of the commodity owned by each.
The solution to such problems are given by Mahāvi¯ra in GSS.[9]
The above sloka means that the sum of the values of conjoint remainders of the commodities be divided by the number of men lessened by 1. The quotient will be the total value of all commodities owned in common. This total value diminished by the specified values gives the value of commodity in each hand.

If x_1, x_2, x_3 and x_4 are the individual parts, using the above rule the solution to the above problem is

$S = x_1 + x_2 + x_3 + x_4 = \frac{22+23+24+27}{4-1} = \frac{96}{3} = 32$

$\therefore\ x_1 = 32-22=10$
$x_2 = 32-23=9$
$x_3 = 32-24=8$
$x_4 = 32-27=5$

By the modern method
$22+x_1 = S$
$23+x_2 = S$
$24+x_3 = S$
$27+x_4 = S$

$\therefore$ Adding all these
$96+(x_1+x_2+x_3+x_4) = 4S$
$\therefore\ 96+S = 4S$
$\therefore\ 3S=96 \rightarrow S=32$
$\therefore\ x_1= 10 \quad x_2=9 \quad x_3=8 \quad x_4=5$

Clearly the values obtained by modern rules are same as the values of Mahāvi¯ra.

Kuttaka - Indeterminate equations of 1st degree

The problem of finding solutions in integers for x and y in an equation of the form ax + c = by where a, b, c are given integers was given great importance by the ancient Indian mathematicians and astronomers. The method of solving such equations is called 'Kuttaka'. But Mahāvi¯ra used the word Kuttākāra. The solution of the 1st degree indeterminate equations in two unknowns is considered as one of the most remarkable contribution of Āryabhata to Mathematics. No other country has originated such an equation and it stands as India's contribution to the world of mathematics.

Mahāvi¯ra in GSS 6.136 $\frac{1}{2}$ gives one method for solving such an equation, which can be explained by a suitable example.

Let the equation be $\frac{63x+7}{23}=y$

```
23) 63 (2
    46
   17) 23 (1
       17
       6) 17 (2
          12
          5) 6 (1
             5
             1) 5 (4
                4
                1
```

First divide the group number 63 by the divisor 23. Then continue the process of division. Here the 1st quotient 2 is discarded. The other quotients 1,2,1,4 are written down in a line one below the other. Then we have to choose such a number as when multiplied by the last remainder 1 and then combined with 7 (the remainder obtained on dividing 7 by 23) will be divisible by the last divisor '1'. So we choose 1 which is written down below the last number and write down the quotient obtained in the above division, with the help of the chosen number, after it. Here we stop. Thus we get the chain of figures 1, 2, 1, 4, 1, 8. Now we multiply the penultimate figure below in the chain i.e., 1 by 4, which is above it and add '8' the last number in the chain. The resulting number '12' is written down in the place corresponding to 4. Then multiplying this '12' by '1' which is the figure just above it in the chain and adding '1' i.e., the figure just below 12 in the series. Write down the result i.e., '13' just above 12. Then multiplying 13 by 2 and adding with 12 gives '38' and write it above 13. Similarly 38x1+13=51 is written above 38. This 51 is divided by 23, (the divisor of the problem) and the remainder '5' is the required value.

Corresponding to this value of x the value of y $=\frac{63\times 5+7}{23}=14$.

Conclusion :

The algebraic techniques presented in the *Gaṇita Sāra Saṅgraha* demonstrate that 9th-century Indian mathematics had already reached a high level of abstraction and procedural clarity. Mahāvīrācārya's methods for solving linear equations—whether through the substitution of unity or the structured resolution of simultaneous systems—reveal a sophisticated understanding of variables and constants. By framing these complex operations within poetic narratives and practical "puzzles," Mahāvīra bridged the gap between abstract logic and human interest. For the contemporary mathematics educator, the G.S.S. serves as more than just a historical relic; it is a testament to a time when mathematics was taught as a vibrant, imaginative, and deeply logical pursuit.

There are many unpublished works of Jainas which are unexplored even now. More studies and researches in Jaina literature may help to reveal new ideas and concepts connected with Mathematics.

References

1. Dr. Padmavathamma, Sri Mahā vi¯ rā cā ryā 's Ganitasā rasangraha, Siddhā ntaki¯ rthi Grandhamā la, Hombuja 2000 v. 3.107, p.133

   रुपं न्यस्याव्यक्ते प्राग्विधिना यत्फलं भवेत्तेन।

   भक्तं परिदृष्टफलं प्रभागजातौ तदज्ञातम्

2. *Ibid*., v.3.108, p.134.

   राशेः कुतश्चिदष्टांशस्त्र्यंशपादोऽर्धपञ्चमः ।

   षष्ठत्रिपादपञ्चांशः किमव्यक्तं फलं दलम्।।

3. *Ibid.,* v. 4.7, p.159

आदायाम्भोरुहाणि स्तुतिशतमुखरः श्रावकस्तीर्थ कृद्भ्यः
पूजां चक्रे चतुर्भ्यो वृषभजिनवरात त्र्यंशमेषाममुष्य ।
त्रयंशं तुर्यं षडंशं तदनु सुमतये तन्नवद्वादशांशौ शेषेभ्यो
द्विद्विपद्मं प्रमुदितमनसादत्त किं तत्प्रमाणम् ॥

4. *Ibid.,* v.4.4, p.157
भागोनरुपभक्तं दृश्यं फलमत्र भागजातिविधौ ।

5 *Ibid.,* v. 6-139½, p.301
ज्येष्ठध्नमहाराशेर्जघन्यफलताडितोनमपनीय।
फलवर्गशेषभागो ज्येष्ठार्घो%न्यो गुणस्य विपरीतम् ।।

6 *Ibid.,* v. 6-270.272½, p.381
दृष्ट्वा कुक्कटयुद्धं प्रत्येकं तौ च कुक्कुटिकौ ।
उक्तौ रहस्यवाक्यैर्मन्त्रौषधशक्तिमन्महापुरुषेण ॥ २७० ॥
जयति हि पक्षी ते मे देहि स्वर्णं ह्यविजयोऽसि दद्यां ते ।
तद्वि त्र्यंशक मद्येत्यपरं च पुनः स संसृत्य || २७१॥
त्रिचतुर्थं प्रतिवाञ्छत्युभयस्माद्वादशैव लाभः स्यात् ।
तत्कुक्कुटिककरस्थं ब्रूहि त्वं गणकमुखतिलक ||

7 *Ibid.,* vv 6.268½ -269½, p.379
स्वस्वच्छेदांशयुती स्थाप्योर्ध्वाधिर्यत: क्रमोत्क्रमशः ।
अन्योन्यच्छेदांशकगुणितौ वज्रापवर्तनक्रमशः ।।
छेदांशक्रमवस्थिततदन्तराभ्यां क्रमेण सम्भक्तौ।
स्वांशहरघ्रान्यहरौ वाञ्छाघ्नौ व्यस्ततः करस्थमितिः ।।

8 *Ibid.,* v. 6-160-162, p.318
वणिजस्ते चत्वारः पृथक् पृथक् शौल्किकेन परिपृष्टाः ।
किं भाण्डसारमिति खलु तत्राहैको वणिक्च्छ्रेष्ठः ॥। १६० ।।
आत्मधनं विनिगृह्य द्वाविंशतिरिरिति ततः परोऽवोचत् ।
त्रिभिरुत्तरा तु विंशतिरथ चतुरधिकैव विंशतिस्तुर्यः ॥ १६१ ॥
सप्तोत्तरविंशतिरिरिति समानसारा निगृह्य सर्वेऽपि ।
ऊचुः किं ब्रूहि सखे पृथक् पृथग्भाण्डसारं मे ||

9 *Ibid.,* v. 6.159, p.317.
रूपोननरैर्विभजेत् पिण्डीकृतभाण्डसारमुपलब्धम्
सर्वधनं स्यात्तस्मादुक्तविहीनं तु हस्तगतम्

Bibliography :

1. B. B. Datta and A. N. Singh, History of Hindu Mathematics Vol.1, Bharati¯ya kala prakāshan, Delhi, 2004
2. L.C. Jain, Mathamatical contents in the digambara jain text of the Karananuyoga group Vol.1 Kundakundajnanapitha Indore 2003
3. Anupam Jain and Sures Chandra Agrawal, Mahāvi¯rācārya, Digambar Jain Thrilok Sodh Samsthan Hasthinapur, Meerut 1985
4. D.E. Smith, History of Mathematics, Vol. 2. Ginn and company- 1925
5. E. T. Bell, Development of Mathematics Macgraw hill, New York: 1940.
6. Padmavathamma, Gaṇitasārasangraha of Mahāvi¯ra -with translation (Kannada and English), Sri Siddhantakirthi Granthamala, Hombuja, Karnataka, 2000,