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## Concepts and calculations in Trilateral geometrical figures as reflected in the- Gaṇitasārasaṃgraha (G.S.S.)

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### Geometry in Ancient India – An Introduction

Kṣetragaṇita or geometry was the foundational inspiration for ancient Indian Mathematics. While computation, arithmetic, and algebra are often highlighted, these fields—along with trigonometry and number theory—largely grew out of geometrical interpretations and constructional methods found in the Sulbasūtras. According to scholar T.A. Sarasvati Amma, the history of Indian geometry spans three eras: Pre-Aryan, Vedic, and Post-Christian. She notes that while early texts like the Sulbasūtras lacked formal proofs, this was because the rationale was preserved through oral tradition or detailed commentaries, many of which were lost during foreign invasions.

Jaina mathematicians significantly advanced the field, integrating complex calculations into their cosmography. Mahāvīra, in the 7th chapter of his *Gaṇita Sāra Saṃgraha*, expanded geometry beyond simple polygons to include specialized shapes such as: Yavākāra (barley-seed shape) Vajrākāra (diamond/thunderbolt shape) Murajākāra (drum shape) and Ṣaṭbhujā kṣetra (hexagon)

By applying these talents to complex curvilinear and combined figures, Jaina scholars like Vīrasena and Śrīdhara ensured geometry remained a central pillar of Indian intellectual history.

### Trilaterals

Three types of trilateral figures are discussed by Mahāvīra in G.S.S. They are equilateral triangle, isosceles triangle and scalene triangle. Mahāvīra considered a triangle as a quadrilateral of one side zero and gave the following rule to find its area.

भुजयुत्यर्धचतुष्काद्भुजहीनाद्धातितत्पदं सूक्ष्मम्।

अथवा मुखतलयुतिदलमवलम्बगुणं न विषमचतुरश्रे॥<sup>1</sup>

In this sloka Mahāvīra says that from the half of the sum of four sides of a quadrilateral subtract each side (separately). Finding the product of such quantities and then taking the square root the minutely accurate measure of the area of the quadrilateral is obtained or the product of the perpendicular from the top to the base and half the sum of the top measure and the base measure will give the minutely accurate area of a quadrilateral.

Since Mahāvīra considered a triangle as a quadrilateral with one side zero the formula for minutely accurate area of a trilateral figure is given as  $\sqrt{s(s-a)(s-b)(s-c)}$ ,

$s = \frac{a + b + c}{2}$ , where a, b, c are the 3 sides. Also area =  $\frac{c}{2} \times p$  where c is one side of the

triangle and 'p' is the perpendicular distance of the opposite vertex from this side. Meanwhile the formula for finding p is also given in GSS 7.49.

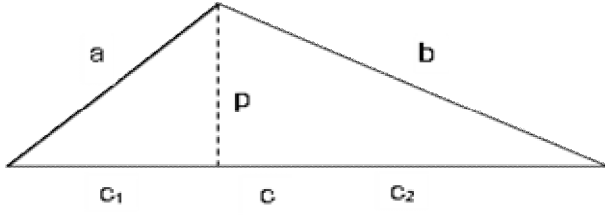
भुजकृत्यन्तरभूहतभूसङ्क्रमणं त्रिबाहुकाबाधे।

तद्धुजवर्गान्तरपदमवलम्बकमाहुराचार्याः॥<sup>2</sup>

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$$\therefore A_a^2 - A_e^2 = \frac{a^4}{16} \quad \therefore a = 2(A_a^2 - A_e^2)^{\frac{1}{4}} \quad \text{and} \quad b = \frac{A_a}{\frac{a}{2}} = \frac{A_a}{(A_a^2 - A_e^2)^{\frac{1}{4}}}$$

So the rule is verified.

Similarly the side<sup>4</sup> of an equilateral triangle of given areas (exact and approximate) is given by Mahāvīra as side  $2(A_a^2 - A_e^2)^{\frac{1}{4}}$

Āryabhatīyam v. 2.6 clearly gives the formula for area of a trilateral figure as  $\text{area} = \frac{1}{2} \times \text{base} \times \text{altitude}$

Brahmagupta also gives the formula<sup>5</sup> for the same as  $s = \frac{a+b+c}{2}$  by treating triangle as a quadrilateral with one side 0. Even though Mahāvīra and Brahmagupta give the formula for area of a triangle, its credit is attributed to Heron of Alexandria<sup>6</sup> But in Yuktidīpika 2.531-534 the area of a triangle is found in a different way by converting it into a quadrilateral and then finding its area.<sup>7</sup>

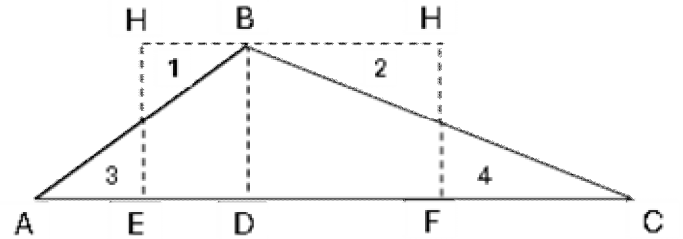


Fig. 3

In fig. 3, triangles (3) and (4) will coincide with (1) and (2) respectively.

$\therefore$  Area of the triangle ABC = Area of the new quadrilateral EFGH

$$= \text{base} \times \text{height} = \frac{1}{2} \text{ base of triangle} \times \text{altitude}$$

In Līlāvati v.- 166 also we can see the same formula for area of a triangle.

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Though Brahmagupta gives an expression for the circumradius of triangle in BrahmasphuTasinddhānta 12-

In the sloka cited above it is given that by doing sankrama<sub>a</sub> between one side (base) and the difference between the squares of the other sides as divided by the base will give the two segments of the base. The square root of the difference between the squares of these segments and of the corresponding adjacent sides gives the measure of the perpendicular *ie.*,

$$c_1 = \frac{1}{2} \left( c + \frac{a^2 - b^2}{c} \right) \quad \text{and} \quad c_2 = \frac{1}{2} \left( c - \frac{a^2 - b^2}{c} \right)$$

$$\text{then } p = \sqrt{a^2 - c_1^2} \text{ or } \sqrt{b^2 - c_2^2}$$

In G.S.S. 7.171 1/2 Mahāvīra gives the rule to find the base and side of an isosceles triangle if the approximate and exact areas of the triangle is known.

**फलवर्गान्तरमूलं द्विगुणं भूत्र्यावहारिकं बाहुः।**

**भूम्यर्धमूलभक्ते द्विसमत्रिभुजस्य करणमिदम् ॥<sup>3</sup>**

In the above sloka Mahāvīra explains that find twice the square root of the difference of the areas (exact and appropriate). This is taken as the base and the approximate area is taken as the side of an isosceles triangle. Dividing the above base and side by half the square root of the above base will give the base and side of the required isosceles triangle.

If  $A_e$  and  $A_a$  are the exact and approximate areas of an isosceles triangle then

$$\text{Base} = \frac{2(A_a^2 - A_e^2)^{\frac{1}{2}}}{(A_a^2 - A_e^2)^{\frac{1}{4}}} = 2(A_a^2 - A_e^2)^{\frac{1}{4}}$$

$$\text{side} = \frac{A_a}{(A_a^2 - A_e^2)^{\frac{1}{4}}}$$

If  $a$  is the base and  $b$  is the side of an isosceles triangle then

$$A_a = \frac{ab}{2} \quad \text{and} \quad A_e = \frac{a}{2} \left( \frac{4b^2 - a^2}{4} \right)^{\frac{1}{2}}$$

$$\text{So } A_e^2 = \frac{a^2}{4} \left( \frac{4b^2 - a^2}{4} \right) = \frac{a^2 b^2}{4} - \frac{a^4}{16} = A_a^2 - \frac{a^4}{16}$$

27, Mahāvīra is the first to speak of a circle inscribed in a triangle and its diameter<sup>8</sup>.

परिधेः पादेन भजेदनायतक्षेत्रसूक्ष्मगणितं तत्  
क्षेत्राभ्यन्तरवृत्ते विष्कम्भोऽयं विनिर्दिष्टः<sup>9</sup>

The above given verse points out that the exact area of any figure other than a rectangle should be divided by one-fourth the perimeter to get the diameter of the circle inscribed within that figure.

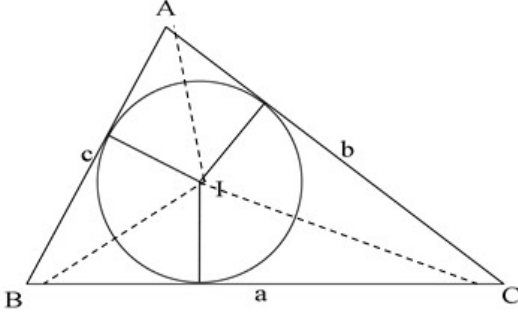


Fig. 4

The rule is quite true because if  $r$  is the radius of the incircle

$$\begin{aligned}
 \text{Area of } \triangle ABC &= \text{Area of } \triangle AIB + \text{Area of } \triangle BIC + \text{Area of } \triangle AIC \\
 &= \frac{1}{2}c.r + \frac{1}{2}a.r + \frac{1}{2}b.r \\
 &= \frac{r}{2}(a+b+c) \\
 \therefore r &= \frac{2\text{Area}}{a+b+c} = \frac{\text{Area}}{\frac{1}{2}(a+b+c)} = \frac{\text{Area}}{\frac{1}{2}\text{Perimeter}} \\
 \therefore \text{diameter} &= \frac{2\text{Area}}{\frac{1}{2}\text{Perimeter}} = \frac{\text{Area}}{\frac{1}{4}\text{Perimeter}}
 \end{aligned}$$

## References

- 1 Sri Mahaviracarya, Ganitasarasangraha translated by Dr. Padmavathamma, Siddhantakirithi Grandhamala, Hombuja 2000 v. 7.50, p.454
- 2 *Ibid.*, v.7-49, p.453
- 3 *Ibid.*, v 7-171½, p.547
- 4 *Ibid.*, v 7-168½, p.546
- 5 C. Krishnan Nambudiri, Bharatiya sastra Cinta, Book-1, Mathematics, Arshaprakasam Prasiddhikarana Samiti, Calicut - 1, 2nd edition, Sept. 2008, p. 217
- 6 T. Heath, A history of Greek Mathematics, Vol. II, Oxford, 1921, p. 103.
- 7 C. Krishnan Nambudiri, *op.cit.*, pp. 209, 210.

8 T. A. Sarasvati Amma Geometry in Ancient and Medieval India, Motilal Banarsidass, Delhi, 1979, p..124.

9 G.S.S. *op. cit.* 7 223 ½, p.585