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Quadratic Equations in Jaina Literature in Sanskrit

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Abstract: Algebra seems to be an interesting field of Ancient Indian mathematicians and they made substantial contributions to this field. The Jaina philosopher mathematicians have on their part made substantial enrichment to the various subdivisions connected with Algebra including linear equations, quadratic equations, system of equations, series and the like. Mahāvi¯ra the great Jaina mathematician has also contributed substantially to this field. He propagated mathematical thoughts into his disciples by very interesting queries and discussions. Different types of equations and their solutions are discussed by Mahāvi¯ra in G.S.S. These ideas of Mahāvi¯ra and his approach in solving such problems can be utilised by the students of modern Mathematics for the better understanding of the concepts. A critical approach on this topic has been attempted in this article.

Jaina mathematicians like Śri¯dhara, Mahāvi¯ra, Vi¯rasena have contributed much to the field of Algebra. Mahāvi¯ra's works contain beautiful and interesting problems rich in mathematical imagination, worth attracting any lay man with a poetic heart. It will be highly helpful for the teachers of Mathematics in making Mathematics easy and interesting as against hard and tiresome. The solution to 1st degree equations in one unknown, 1st degree equations in two or more unknowns, quadratic equations, system of equations etc.. are discussed by Mahāvi¯ra in his pure Mathematics work Ganita Sāra Saṅgraha (G.S.S) written in Sanskrit. Some of the significant ideas discussed in connection with quadratic equations and their solutions can be classified as given below

a) **Type 1**

When expressing the rules of problems on fractions in G.S.S, Mahāvi¯ra mentions:

मूलार्धाग्रे छिन्द्यादंशोनैकेन युक्तमूलकृतेः
दृश्यस्य पदं सपदं वर्गितमिह मूलजातौ स्वम् ।। [1]

As per the above sloka half of the coefficient of the (square root of the unknown quantity) and then the known remainder should be divided by one as diminished by (the fractional coefficient of the unknown quantity). The square root of the sum of the known remainder so treated as combined with the square of the coefficient of the square root of the unknown quantity similarly treated and then associated with the similarly treated coefficient of the (square root of the unknown quantity) and squared as a whole gives rise to the required unknown quantity. Which means algebraically if $x - bx - c\sqrt{x} - a = 0$ (which is the algebraic expression of mūla variety problems),

$$x=\left[\frac{\frac{c}{2}}{1-b}+\sqrt{\frac{a}{1-b}+\left(\frac{\frac{c}{2}}{1-b}\right)^{2}}\right]^{2} \quad \}$$

Taking $\sqrt{x}=t$, the given equation becomes $(1-b)t^2 - ct - a = 0$ By putting $1-b=k$, it becomes $kt^2 - ct - a = 0$

$$\therefore t = \frac{c \pm \sqrt{c^2+4ka}}{2k}$$

$$= \frac{\frac{c}{2}}{k} + \sqrt{\left(\frac{\frac{c}{2}}{k}\right)^2 + \frac{a}{k}} = \frac{\frac{c}{2}}{1-b} + \sqrt{\frac{a}{1-b} + \left(\frac{\frac{c}{2}}{1-b}\right)^2}$$

$$\therefore \mathbf{x} = \mathbf{t^2}$$

So this rule is true according to the modern mathematics.

In GSS 4-34 he has given an example[2] to illustrate the rule "One-fourth of a herd of camels was seen in the forest, twice the square root had gone to mountain slopes and 3 times 5 camels were found to remain on the bank of a river. What is the numerical measure of that herd of camels?"

Algebraically the given problem can be written as if x is the number of camels and $\mathbf{x - \frac{1}{4}x - 2\sqrt{x} - 15 = 0}$, then x=?.

Comparing this equation with $\mathbf{x - bx - c\sqrt{x} - a = 0}$ we get $b=\frac{1}{4}$, $c=2$, $a=15$

Then by the given rule

$$x = \left[\frac{2/2}{1-1/4} + \sqrt{\frac{15}{1-1/4} + \left(\frac{2/2}{1-1/4}\right)^2}\right]^2 = \left[\frac{1}{3/4} + \sqrt{\frac{15}{3/4} + \left(\frac{1}{3/4}\right)^2}\right]^2$$

$$= \left[\frac{4}{3} + \sqrt{5\times 4 + \frac{16}{9}}\right]^2$$

$$= \left[\frac{4}{3} + \sqrt{\frac{180+16}{9}}\right]^2$$

$$= \left[\frac{4}{3} + \sqrt{\frac{196}{9}}\right]^2$$

$$= \left(\frac{4}{3} + \frac{14}{3}\right)^2 = \left(\frac{18}{3}\right)^2$$

$$= 6^2 = 36$$

The same rule is given by Śrīdhara in Patīgaṇita v-76.

b) Type 2

In GSS 4.41 Mahāvīra, gives another problem:-[3]

One third of a herd of elephants and 3 times the square root of the remaining part where seen on a mountain slope and in a lake was seen a male elephant along with 3 female elephants. How many were the elephants?

This problem can be written algebraically as $x - \frac{1}{3}x - 3\sqrt{x - \frac{1}{3}x} - 4 = 0$

i.e., $x - bx - c\sqrt{x - bx} - a = 0$ where $b=\frac{1}{3}$, $c=3$, $a=4$

The rule to solve such a problem is given by Mahāvīra as

पददलवर्गयुताग्रान्मूलं सप्राक्पदार्धमस्य कृतिः।

दृश्ये मूलं प्राप्ते फलमिह भागं तु भागजा[4]

The above stanza means that find the square of, half the coefficient of the square root of the remaining part of the unknown collective quantity and combine it with the known number remaining. Then extract the square root of this sum and combine it with half of the previously mentioned coefficient of the square root of the remaining part of the unknown collective quantity. The square root of this will be the required result.

By this rule

$$x - bx = \left[\frac{c}{2} + \sqrt{\left(\frac{c}{2}\right)^2 + a}\right]^2$$

Using this rule $x - \frac{1}{3}x = \left[\frac{3}{2} + \sqrt{\left(\frac{3}{2}\right)^2 + 4}\right]^2$

$$= \left[\frac{3}{2} + \sqrt{\frac{9+16}{4}}\right]^2$$

$$= \left(\frac{3}{2} + \frac{5}{2}\right)^2 = 4^2 = 16$$

$$\therefore x = 16 \times \frac{3}{2} = 24$$

c) Type 3

In GSS 4-67 Mahāvi¯ra puts a problem.[5] "The sum of 2 quantities which are respectively equivalent to the square roots of the (whole) collection of pigeons and of (that same) collection as diminished by the cube of 4 amounts to 16. How many are the birds in that collection".

The above problem can be written algebraically as $\sqrt{x} + \sqrt{x - 4^3} = 16$, i.e., $\sqrt{x} + \sqrt{x - 64} = 16$ A rule to solve such an equation is given by as.

मिश्रकृतिरूनयुक्ता व्यधिका च द्विगुणमिश्रसम्भक्ता।

वर्गीकृता फलं स्यात्करणमिदं मूलमिश्रविधौ।।[6]

As per the sloka, to the square of the combined sum the given minus quantity is added or the given plus quantity is subtracted. Then this quantity is divided by twice the combined sum. when squared gives the required value.

So If $\sqrt{x} + \sqrt{x \pm d} = m$ then $x = \{(m^2 \mp d)/2m\}^2$

Here in the above problem m =16 and d = 64 so that

$$X = \{(16^2 + 64)/2x16\}^2 = 16^2 \text{ x } \frac{(16+4)^2}{2^2 \times 16^2}$$

$$= \frac{2^2}{2^2} = 100$$

d) Type 4

When discussing the miscellaneous problems on fractions Mahāvi¯ra gives a rule to solve equations of the form $\frac{a}{b}x^2 - x + c - 0$

स्वांशाप्तहरादूनाच्चतुर्गुणाग्रेण तद्धरेण हतात्।

मूलं योज्यं त्याज्यं तच्छेदे तद्दलं वित्तम्।।[7]

This sloka means that divide the denominator of the fractional part of the unknown quantity by its own numerator. Subtract four times the given known part of the quantity from the above result. Multiply this by that same denominator (dealt with as above). The square root of this is to be added as well as subtracted from the denominator. The half of this is the collective quantity.

An example to this rule is given in 4.59 which is given below

$\frac{1}{16}$ part of a collection of peacocks as multiplied by itself was found on a mango tree, $\frac{1}{9}$ of the remainder as multiplied by that same and remaining 14 were found in a grove of tamala trees. How many are they in all?

Here given that $\frac{x}{16}.\frac{x}{16} + \frac{15x}{16}.\frac{1}{9}.\frac{15x}{16}.\frac{1}{9} + 14 = x$ where 'x' is the number of peacocks.

i.e. $\frac{x^2}{16^2} - \frac{225x^2}{16^2 x 9^2} + 14 - x$

i.e., $\frac{(81+225)x^2}{256\ x\ 81} + 14 - x$

i.e., $\frac{306}{20736}x^2 - x + 14 = 0$

$\frac{17}{1152}x^2 - x + 14 = 0$

which is of the form $\frac{a}{b}x^2 - x + c = 0$

As per the above rule, $x = \frac{b}{a} + \sqrt{\left(\frac{b}{a} - 4c\right)\frac{b}{a}}$.

But according to modern mathematics. $x - \frac{b}{2a} \pm \frac{\sqrt{\left(\frac{b}{a} - 4c\right)\frac{b}{a}}}{2}$

So the solution of Mahāvi¯ra cannot be accepted

But Śri¯dhara gives the solution[8] of $ax^2 + bx = c$ as

चतुराहत वर्गसमै रूपैः पक्षद्वयं गुणयेत्

अव्यक्त वर्गरुपैः युक्तौ पक्षौ ततो मूलं

This sloka states that multiply the coefficient of square of unknown by 4 and multiply both the sides by this product. Then add the square of coefficient of the unknown on both sides and find the square root.

Consider $ax^2 + bx = c$ where x is the unknown , 'a' is the coefficient of x^2 and 'b' is the coefficient of x. Then by the rule

$ax^2 + bx = c$ **................(1)**

i.e., $(1)\ x\ 4a \rightarrow 4a^2x^2 + 4abx = 4ac$ **...............(2)**

$(2) + b^2 \rightarrow 4a^2x^2 + 4abx + b^2 = 4ac + b^2$

$\rightarrow (2ax + b)^2 = 4ac + b^2$

$\rightarrow 2ax + b = \sqrt{4ac + b^2}$ **So x** $= \frac{\sqrt{4ac \mid b^2} \quad b}{2a}$

which is exactly the modern method.

Śri¯pati solved the same equation in another way. By his rule,[9] multiply both sides by the coefficient of the square of the unknown and add square of half the coefficient of the unknown. Then take the square root.

i.e $ax^2 + bx = c$

i.e $a^2x^2 + abx = ac$ **i.e** $a^2x^2 + abx + \left(\frac{b}{2}\right)^2 = ac + \left(\frac{b}{2}\right)^2$

i.e $(ax + \frac{b}{2})^2 = ac + \left(\frac{b}{2}\right)^2$

$$ax + \frac{b}{2} = \sqrt{ac + \left(\frac{b}{2}\right)^2}$$

$$x = \frac{\sqrt{ac + \left(\frac{b}{2}\right)^2} - \frac{b}{2}}{a}$$

e) Type 5

In Pati¯gaṇita v.74. Śri¯dhara gives the solution of equations of the form $x \pm a\sqrt{x} = b$ By this rule $x = \left[\sqrt{b + \left(\frac{a}{2}\right)^2} \mp \frac{a}{2}\right]^2$. This rule is obtained by the usual rule for solving an equation reducible to a quadratic form by completing the square. The same rule is given by Bhāskara in Li¯lāvati 62-63.

Indians were aware of the 2 solutions of quadratic equations Bhāskara explained that even though a quadratic equation has 2 solutions no need to take both the solutions since it creates some problems. Brahmagupta also accepted only one solution avoiding the other citing some problems arising in practical cases. Mahāvi¯ra accepted only the positive solutions and avoided negative solutions because he was treating only practical problems.

Foot note

1. Dr.Padmavathamma, Sri Mahāvi¯rācāryā's Ganitasārasangraha , Siddhāntaki¯rthi Grandhamāla, Hombuja 2000 , v. 4-33, p.169
2. *Ibid.*, v. 4-34, p.170
3. *Ibid.*, v. 4-41.p.173
4. *Ibid.*, v. 4-40, p.172
5. *Ibid.*, v. 4-67, p.189
6. *Ibid.*, v. 4-65, p.187.
7. *Ibid.*, v. 4-57, p. 183
8. Ramachandra Menon, Bharatiyaganitam, Kerala Bhasha Institute Thiruvananthapuram 1989 , pp. 178-179.
9. *Ibid.*, p. 179.